

Sideband-Aided Receiver Arraying

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Recently, there has been a substantial effort to increase the amount of data that can be received from outer planet missions by coherently combining signals from ground antennas in such a way as to increase the total effective aperture of the receiving system. However, as these signals become weaker, the baseband arraying technique in current use degrades somewhat due to carrier jitter. One solution to this problem is Sideband-Aided Receiver Arraying (SARA). In SARA, sidebands demodulated to baseband in a master receiver at the largest antenna are used to allow slave receivers in the other antennas to track the sideband power in the signal rather than the carrier power. The already existing receivers can be used in the slaves to track and demodulate the signals in either a residual carrier or a suppressed carrier environment. The resultant baseband signals from all the antennas can then be combined using existing baseband combining equipment. Computer simulations of SARA show increases in throughput (measured in data bits per second) over baseband-only combining of 17% at Voyager 2 Uranus encounter and 31% at Neptune for a four-element antenna array and (7, 1/2) convolutional coding.

I. Introduction

Many different antenna arraying systems have been proposed for receiving increased telemetry rates from outer planet missions. The performance of two of these schemes, "baseband-only combining" (BBO) and "baseband combining with combined carrier referencing" (CCR), have been studied by the authors (Ref. 1). BBO (Refs. 2, 3) is the scheme that is currently used by the DSN for Voyager. In BBO each receiver demodulates its own carrier using a conventional phase locked loop (PPL). CCR schemes involve summing the carrier signals from the arrayed antennas in phase so that the array behaves like a single antenna with aperture equal to the sum of the apertures of the individual array elements. Two different implementations of CCR have been studied in detail. These are the "master-slave system" (Refs. 4, 5) and "virtual center

arraying" (Ref. 6). All of the above-mentioned arraying schemes require the presence of a residual carrier for tracking.

Sideband-Aided Receiver Arraying (SARA) is an arraying system that offers increased performance over BBO and CCR at high modulation indices (such as those proposed for Voyager 2 Uranus and Neptune encounters) and the ability to operate in a suppressed carrier environment. Suppressed carrier transmission, in which the modulation index is equal to 90° , offers a $\sec^2(\theta)$ improvement in bit energy-to-noise ratio over residual carrier transmission with a modulation index of θ . For Voyager 2 Uranus and Neptune encounters, suppressing the carrier will yield a 0.25-dB improvement in E_b/N_0 .

SARA is a master-slave type system. Either a conventional tracking loop or suppressed carrier tracking loop may be used

in the master channel to demodulate the carrier and produce good data and subcarrier estimates. A suppressed carrier loop will yield better performance in most cases, and the conceptual block diagrams described in this article are for this type of loop. The master channel is associated with the largest antenna in the array where the signal-to-noise ratio is largest. The slave receivers contain conventional tracking loops that are aided by the subcarrier and data estimates derived in the master. These sidebands are relatively low frequency and low bandwidth signals that may be transmitted digitally over existing microwave links. Hardware must be provided in each slave for proper alignment of its modulation with the estimates it receives from the master. This aiding allows the slave receivers to track the sideband power in the incoming signal (rather than the carrier power), with losses that are lower than those that would be present in systems with separate suppressed carrier loops in each slave. The demodulated baseband signals from the master and all slaves are then combined in a baseband combiner in the same way as in BBO.

In the following sections, SARA is described in more detail. Loop signal-to-noise ratios are derived for the master and slave receivers. Degradations due to subcarrier demodulation, subcarrier alignment, and data alignment are discussed. Section VI discusses the performance of SARA at Voyager 2 Uranus and Neptune encounters for two-, three-, and four-element arrays as determined from computer simulations. These results show that SARA, operating in a suppressed carrier environment, has less radio loss than either BBO or CCR. Graphs of data rate vs time for the two encounters are also presented, and these clearly show the increased telemetry capability that SARA allows.

II. The Data-Aided Loop

In the preferred implementation of SARA, the master receiver should be capable of operating with both residual and suppressed carrier signals. Three well-known regenerative loops can be employed for obtaining a carrier reference from a suppressed carrier signal. These are the squaring loop (Ref. 7), the Costas loop (Ref. 7), and the "data-aided" loop (Ref. 8). Moreover, these implementations provide very good carrier reference estimates from residual carrier signals whose carrier power is suppressed by more than 12 dB below the data sideband power (Ref. 9). In the body of this article, only the data-aided implementation will be considered. Comparative analyses of data-aided and Costas or squaring loops are given in Refs. 7 and 8. A block diagram of a data-aided loop appears in Fig. 1. The input to this loop is assumed to be a phase-modulated sinusoid of frequency ω_0 with no subcarrier. Since the data-aided loop tracks only the data power in the signal, it may be assumed that $x(t)$ has the form

$$\begin{aligned} x(t) &= \sqrt{2P_D} \sin [\omega_0 t + \pi/2 d(t) + \phi] + n_1(t) \\ &= \sqrt{2P_D} d(t) \cos (\omega_0 t + \phi) + n_1(t) \end{aligned}$$

where P_D is the data power of the signal, $d(t)$ is the data in the form of plus and minus ones that are constant for a symbol time T_s , and ϕ is the phase of the signal. In this analysis it is assumed that ϕ changes slowly with time compared to $d(t)$. The noise, $n_1(t)$, is assumed to be a narrowband Gaussian process with one-sided spectral density N_0 W/Hz in the region of interest.

The output of the VCO is

$$r(t) = \sqrt{2} K_1 \cos (\omega_0 t + \hat{\phi})$$

where K_1 is a gain associated with the VCO and $\hat{\phi}$ is the loop's estimate of the phase error ϕ . It follows that, after filtering out the double frequency terms,

$$y(t) = \sqrt{P_D} K_1 K_2 d(t) \cos (\phi - \hat{\phi}) + n_2(t)$$

where K_2 is the gain of the upper mixer and n_2 is a white Gaussian process with one-sided spectral density $K_1 K_2 N_0$ W/Hz over the closed-loop bandwidth. The data detector integrates $y(t)$ over a symbol time T_s . The sign of this integral defines the data estimate

$$e(t) = \hat{d}(t - T_s).$$

The probability that a correct estimate is made is

$$pr(\hat{d} = d) = 1/2 + 1/2 \operatorname{erf} (\sqrt{E_s/N_0} \cos [\phi - \hat{\phi}])$$

where

$$\operatorname{erf}(x) = 2/\sqrt{\pi} \int_0^x \exp(-t^2) dt.$$

Therefore, the mean value of $e(t)$ is given by

$$\begin{aligned} \bar{e}(t) &= d(t - T_s) pr(\hat{d} = d) - d(t - T_s) pr(\hat{d} \neq d) \\ &= d(t - T_s) \operatorname{erf} (\sqrt{E_s/N_0} \cos [\phi - \hat{\phi}]). \end{aligned}$$

The quadrature carrier estimate $s(t)$ is given by

$$s(t) = -\sqrt{2} K_3 \sin (\omega_0 t + \hat{\phi})$$

where K_3 is a gain associated with the VCO and the quadrature generator box (labeled " $\pi/2$ " in the figure). After the lower mixer, an associated low-pass filter to attenuate the double frequency terms, and a one symbol time delay,

$$w(t) = \sqrt{P_D} K_3 K_4 d(t - T_s) \sin(\phi - \hat{\phi}) + n_3(t)$$

where K_4 is the gain in the mixer and $n_3(t)$ is a white Gaussian noise process with one-sided spectral density $K_3 K_4 N_0$ W/Hz. Notice that n_2 and n_3 are uncorrelated due to the fact that they are in quadrature with one another.

Under the previously stated assumption that ϕ varies slowly with respect to $d(t)$,

$$\begin{aligned} z(t) &\approx \bar{e}(t) w(t) \\ &= \sqrt{P_D} K_3 K_4 \sin(\phi - \hat{\phi}) \operatorname{erf}[\sqrt{E_s/N_0} \cos(\phi - \hat{\phi})] + n_4(t). \end{aligned}$$

For small phase errors $\phi - \hat{\phi}$ this linearizes to

$$z(t) = \sqrt{P_D} K_3 K_4 [\phi - \hat{\phi}] \operatorname{erf}(\sqrt{E_s/N_0}) + n_4(t)$$

from which the standard PLL theory (see Appendix A) shows the loop signal-to-noise ratio (loop SNR) of the data-aided loop to be

$$\rho = \frac{P_D}{N_0 B_L} \operatorname{erf}^2(\sqrt{E_s/N_0})$$

where B_L is the one-sided closed-loop bandwidth of the receiver.

III. The Slaves

In SARA, the modulation estimates obtained in the master are sent to each slave to aid in tracking. Figure 2 shows a diagram of a slave that could be used in conjunction with the data-aided loop described in the last section. In this implementation, a hard-limited estimate of the data ($e(t)$ in Fig. 2) is passed to the slave to derive a carrier reference. In Appendix B, the rationale for this procedure is developed. The input signal is again assumed to have the form

$$x_s(t) = \sqrt{2P_{Ds}} d(t) \cos[\omega_0 t + \phi_s] + N_1(t) \quad (1)$$

where P_{Ds} is the data power in the slave and ϕ_s is the phase of the signal. It is assumed that ϕ_s changes slowly with respect to $d(t)$ although it is not necessarily the same as ϕ . The carrier estimate from the VCO is

$$r_s(t) = -\sqrt{2}K \sin[\omega_0 t + \hat{\phi}_s]$$

where K is a gain associated with the VCO and $\hat{\phi}_s$ is the phase estimate of the slave loop. After filtering out the double frequency components and delaying, it follows that

$$y_s(t) = \sqrt{P_{Ds}} K K' d(t - T_s) \sin(\phi_s - \hat{\phi}_s) + N_2(t) \quad (2)$$

If $e(t)$ is perfectly aligned, then

$$\begin{aligned} z_s(t) &\approx \bar{e}(t) y_s(t) \\ &= \sqrt{P_{Ds}} K K' \sin(\phi_s - \hat{\phi}_s) \operatorname{erf}[\sqrt{E_s/N_0} \cos(\phi - \hat{\phi})] \\ &\quad + N_3(t) \end{aligned}$$

where N_3 is a white Gaussian process independent of the noise in the master. If both the phase errors in the master and the slave are small, then this can be linearized to

$$z_s(t) = \sqrt{P_{Ds}} K K' [\phi_s - \hat{\phi}_s] \operatorname{erf}(\sqrt{E_s/N_0}) + N_3(t).$$

Standard PLL theory (see Appendix A) then shows that the loop SNR in the slave is given by

$$\rho_s = \frac{P_{Ds}}{N_{0s} B_{Ls}} \operatorname{erf}^2(\sqrt{E_s/N_0}) \quad (3)$$

where N_{0s} is the one-sided noise spectral density in the slave, and B_{Ls} is the one-sided closed-loop bandwidth of the slave.

IV. Subcarrier Demodulation

In Sections II and III, only signals without subcarriers were considered. If the incoming signal has a subcarrier, then a subcarrier tracking loop must be added to the master receiver in order to demodulate the subcarrier before the data can be detected. The subcarrier estimate must also be sent to the slaves along with the data estimate. Figure 3 shows one possible configuration for a SARA master and slave with such a subcarrier loop added. The subcarrier is assumed to be a square wave in the following analysis.

The subcarrier tracking loop in the master is, itself, a data-aided loop similar to that described in Section II. It shares the data detection arm with the carrier tracking loop. The principal difference is that the input to the subcarrier loop is an amplitude-modulated square wave,

$$i(t) = \sqrt{P_D} K_1 K_3 d(t) \sin(\omega_{sc} t) \cos(\phi - \hat{\phi}) + n_2(t)$$

where P_D , K_1 , K_3 , ϕ , $\hat{\phi}$, and n_2 are as defined in Section II. The subcarrier frequency ω_{sc} is assumed to be high compared to the channel symbol rate $R_s = 1/T_s$. For Voyager 2 Uranus and Neptune encounters, $\omega_{sc} = 360$ kHz while $R_s < 40$ kHz. Recall that ϕ and $\hat{\phi}$ also vary slowly compared to $d(t)$. Under these assumptions, the analysis of the subcarrier tracking loop is identical to that of the carrier tracking loop with the exception that only the first harmonic of the subcarrier is retained after mixing with the subcarrier estimate $j(t)$. Since the fraction of the total square wave power in the first harmonic is $8/\pi^2$, the subcarrier tracking loop SNR is given by

$$\rho_{sc} = \frac{8}{\pi^2} \frac{P_D}{N_0 B_{Lsc}} \text{erf}^2(\sqrt{E_s/N_0})$$

where B_{Lsc} is the closed-loop bandwidth of the subcarrier tracking loop.

In practice, once subcarrier and carrier lock have occurred in the master, the bandwidth of the subcarrier loop can be narrowed, since most of the doppler error in the incoming signal is removed by carrier demodulation. Hence, in the steady state, subcarrier demodulation losses should be small compared to carrier demodulation losses in the master receiver.

In order to aid the slave receivers, the subcarrier and data estimates are recombined. The subcarrier estimate $j(t)$ must be delayed by one symbol time before being mixed with the data estimate. The slave is then aided just as in Section III, with both data and subcarrier demodulation occurring simultaneously.

V. Alignment Losses

Thus far it has been assumed that the modulation estimates from the master are perfectly aligned with the modulation in the slaves. In practice, there will be losses resulting from an imperfect alignment of these signals. This misalignment will result in a degradation in the loop SNR of the slaves and therefore will result in poorer performance in those receivers.

Suppose that the modulations of the master and slave are misaligned by τ seconds (where $|\tau| < 1/4\omega_{sc}$). Then the average value of the product of the subcarriers will not be one as in the case of perfect alignment. Instead it will be

$$\begin{aligned} \overline{\sin(\omega_{sc}t) \sin(\omega_{sc}t + \tau)} &= (1/(2\omega_{sc}) - 2|\tau|)/(1/(2\omega_{sc})) \\ &= 1 - 4\omega_{sc}|\tau|. \end{aligned}$$

If the transition probability in the symbol stream $d(t)$ is p_t , then the average value of the product of the data signals is

$$\begin{aligned} \overline{d(t) d(t + \tau)} &= (T_s/p_t - 2|\tau|)/(T_s/p_t) \\ &= 1 - 2p_t R_s |\tau|. \end{aligned}$$

These lead to a degraded slave loop SNR of

$$\rho_s = \frac{P_D}{N_0 B_{Ls}} \text{erf}^2(\sqrt{E_s/N_0}) (1 - 4\omega_{sc}|\tau|)^2 (1 - 2p_t R_s |\tau|)^2.$$

Graphs of the losses due to improper subcarrier and data alignment in the slaves appear in Figs. 4 and 5 respectively. A subcarrier frequency of 360 kHz and a data frequency of 38.4 kps were assumed in these plots. These are typical values for the Voyager 2 Uranus encounter. Notice that the alignment loss is much greater in the subcarrier than in the data.

The primary cause of misalignment in SARA will be inter-receiver timing problems due to link jitter. A study of the existing GCF10-DSS 14 microwave link (Ref. 10) shows a two-way link jitter on the order of 20 nanoseconds. With additional alignment hardware in the slaves to take care of gross interreceiver time delays, the alignment degradations due to the link can probably be kept to a tolerable level (less than a dB of loop SNR).

VI. Numerical Results

The performance of SARA on coded telemetry was determined by software simulation. The code used was the DSN standard (7, 1/2) convolutional code with Viterbi (maximum likelihood) decoding. Perfect subcarrier demodulation, symbol tracking, and modulation alignment were assumed. The only losses were the space loss, a 90% weather confidence attenuation, and the independent carrier phase jitter in each receiver.

Carrier loop simulation software was used to generate the phase errors for each of n receivers according to the distribution

$$p_i(\phi_i) = \frac{\exp(\rho_i \cos \phi_i)}{2\pi I_0(\rho_i)} \quad (i = 1, 2, 3, \dots, n)$$

where ϕ_i is the carrier phase error in the i^{th} receiver, ρ_i is the loop SNR of that receiver and I_0 is the zero-order Bessel function. The loop SNRs for SARA master and slave receivers are those given in Sections II and III respectively. If a signal with total bit signal-to-noise ratio E_b/N_0 is received by this

array, then the bit SNR seen at the output of the baseband combiner is

$$TOTSNR = E_b/N_0 \left(\sum_{i=1}^n w_i \cos \phi_i \right)^2$$

where w_i is the fraction of signal amplitude seen by the i th antenna. A software Viterbi decoder was used to monitor the decoded bit errors made by the system.

The performances of BBO and CCR systems were also determined by the software. BBO was simulated in exactly the same way as SARA except that the ρ_i were the loop SNRs of conventional receivers. CCR was modelled as a single antenna whose aperture would receive the total power incident on the array.

The power-to-noise ratios for Voyager 2 Uranus encounter were taken from design control tables (Ref. 11). The power-to-noise ratios for Neptune encounter were assumed to be 3.5 dB less than those for Uranus encounter due to the increased distance of the spacecraft from Earth. The loop SNRs of the various receivers were calculated from these power-to-noise ratios according to the formulas given in Ref. 1 and in Sections II and III. A threshold loop bandwidth of $2B_{L0} = 30$ Hz was assumed in all receivers for comparison purposes.

Tables 1 and 2 show the relative performance of BBO, CCR, and SARA at Voyager 2 Uranus and Neptune encounters for two-, three-, and four-element arrays. The modulation indices were optimized to within 2 degrees. The numbers labeled "maximum data rate" represent the highest possible information bit rate that would allow transmission at a bit error rate of 5×10^{-3} . Since, in practice, Voyager has only a small number of available data rates, these numbers should be considered as a convenient measure of system performance rather than as actual data rates. From the tables it is clear that SARA performs substantially better than CCR and BBO for these Voyager scenarios. Data rates for SARA are about 11% higher than either of the other schemes.

The performance of SARA was also determined at a modulation index of 76° . The maximum data rates for Uranus and Neptune encounters were found to be 26.5 kbps and 11.7 kbps respectively. This means that data rate increases of 2.3% for Uranus encounter and 5.4% for Neptune encounter over CCR are attainable with SARA without suppressing the carrier.

The relative performance of the various arraying schemes is also shown in plots of maximum data rate vs time for the two

Voyager planetary encounters. These plots appear in Fig. 6 for Uranus encounter and in Fig. 7 for Neptune encounter. (In the case of Neptune encounter, the time axis should be read as hours relative to some unspecified zero hour.) Horizontal dotted lines represent the data rates that are actually available on Voyager. One may determine the number of hours of transmission at an average bit error rate of 5×10^{-3} from these graphs for each of the arraying schemes, data rates, and array configurations. This is done by observing how much of the particular data rate curve is above the data rate line in question. For example, with a four-element array at Uranus encounter, SARA provides about 13.8 hours of viewing at a data rate of 19.2 kbps, while CCR allows only about 12.3 hours and BBO only 11.8 hours. This represents an improvement in throughput of 12% over CCR and 17% over BBO. At Neptune, the corresponding improvements at 8.4 kbps are 12% over CCR and 31% over BBO.

VII. Summary and Conclusions

The numerical results presented in Section VI show that SARA offers improvements in throughput for Voyager 2 Uranus and Neptune encounters. The improvements are due primarily to two effects. The first is the ability of SARA to work in a suppressed carrier environment. For example, while the optimum modulation index for CCR at Uranus encounter is about 76° , SARA can operate at a modulation index of 90° . Consequently, SARA can see 0.26 dB (6%) more data power than CCR. The second effect is the smaller radio loss of SARA as compared to the other two systems. The radio loss is lower because the sideband aiding in both the master and the slaves greatly increases the receiver loop SNRs over conventional phase lock loops. A greater loop SNR means smaller phase jitter and better tracking.

Another advantage to SARA is that it can be made compatible with existing DSN hardware. The slave receivers can be existing Block III or Block IV receivers with a mixer added for the sideband aiding. The master can be built as described in the text, or it can be made out of an existing receiver by taking the modulation estimates out of the Subcarrier Demodulation Assembly (SDA) and the Symbol Synchronizer Assembly (SSA). The modulation can be sent to the slave receivers over existing microwave links, and they can be aligned with the modulations in the slaves using reprogrammed Real Time Combiners (RTCs). One possible DSN implementation of SARA is shown in Fig. 8.

A number of issues associated with the SARA concept remain to be addressed. Optimal loop filter parameters for specific situations such as the Voyager planetary encounters should be derived. The question of tracking loop acquisition is also very important. The sideband aiding in the slaves should

help them acquire in a low SNR environment where a conventional PLL might not lock at all. The master has three coupled loops: the carrier loop, subcarrier loop, and the data loop (included in the data detection function). An efficient lockup

sequence should be established. It may be necessary to change filter parameters for the acquisition process. Also, a device for detecting when lock has occurred in each loop might be needed.

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Table 1. Performance of various arraying schemes at Voyager 2 Uranus encounter (0 hour GMT, Day 34, 1986; BER = 0.005, $2B_{LO}$ = 30 Hz)

Array configuration		Optimal modulation index, deg	Radio loss, dB	Maximum data rate, kbps
64m-34m(T/R):				
	BBO	74	0.32	16.1
	CCR	74	0.25	16.4
	SARA	90	0.07	18.3
64m-34m(LO)-34m(T/R):				
	BBO	74	0.29	20.9
	CCR	74	0.22	21.3
	SARA	90	0.10	23.4
64m-34m(LO)-34m(LO)-34m(T/R):				
	BBO	74	0.30	25.1
	CCR	76	0.22	25.9
	SARA	90	0.08	28.3

Table 2. Performance of various arraying schemes at Voyager 2 Neptune encounter (Peak of day curve; BER = 0.005, $2B_{LO}$ = 30 Hz)

Array configuration		Optimal modulation index, deg	Radio loss, dB	Maximum data rate, kbps
64m-34m(T/R):				
	BBO	72	0.51	6.75
	CCR	72	0.45	6.84
	SARA	90	0.25	8.04
64m-34m(LO)-34m(T/R):				
	BBO	72	0.59	8.53
	CCR	72	0.34	9.04
	SARA	90	0.20	10.2
64m-34m(LO)-34m(LO)-34m(T/R):				
	BBO	70	0.43	10.4
	CCR	74	0.36	11.1
	SARA	90	0.23	12.2

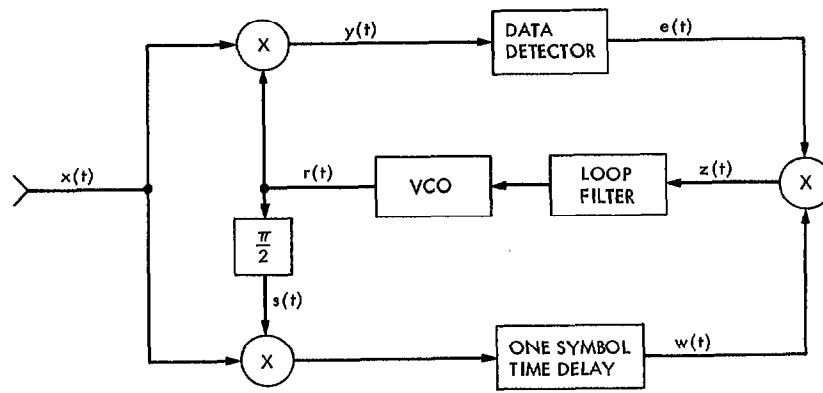


Fig. 1. The data-aided loop

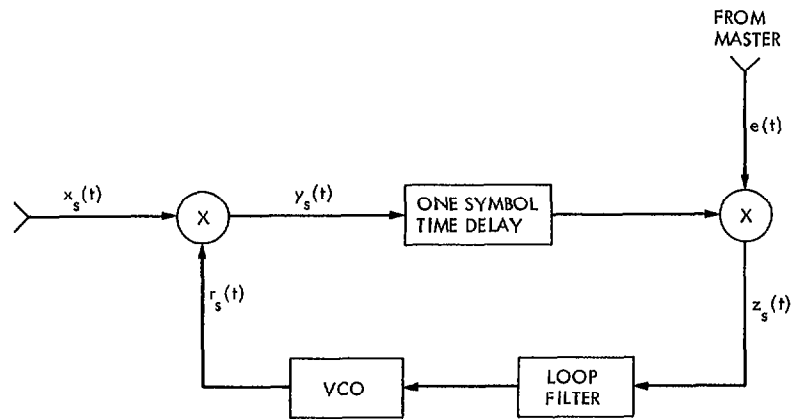


Fig. 2. A slave receiver in SARA

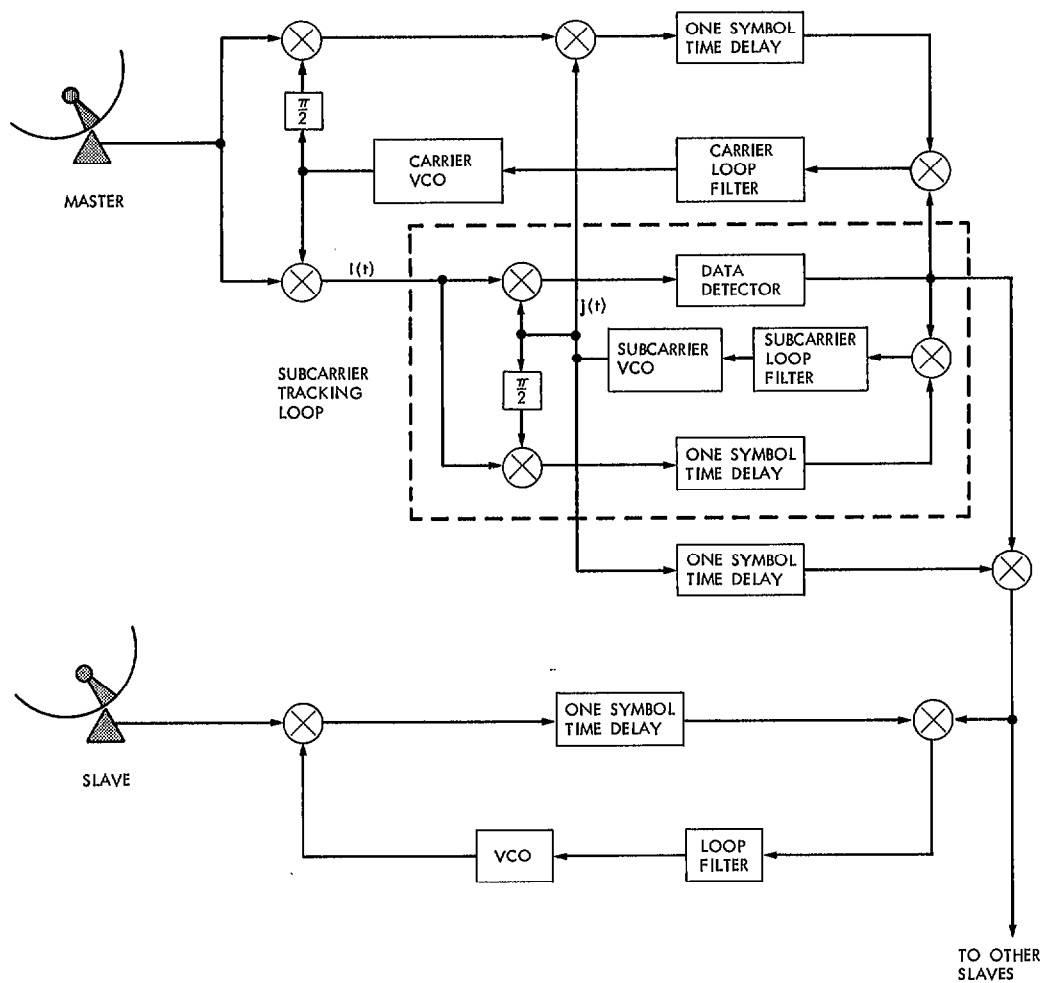


Fig. 3. SARA with subcarrier demodulation

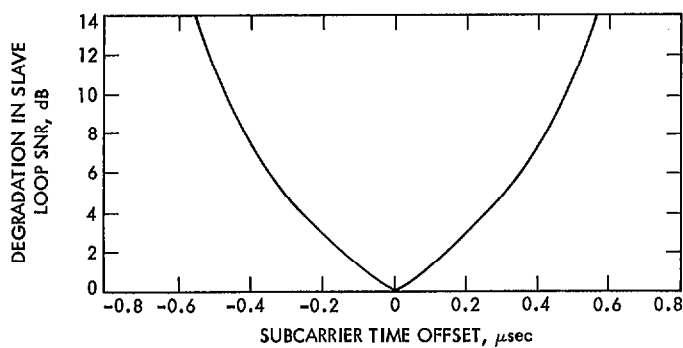


Fig. 4. Degradation in loop SNR of SARA slave due to misalignment of subcarriers ($\omega_{sc} = 360 \text{ kHz}$)

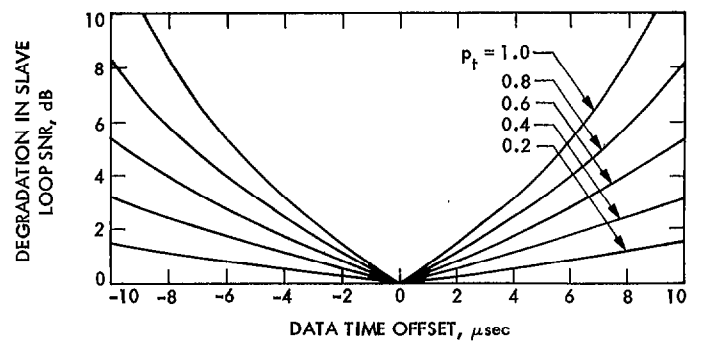


Fig. 5. Degradation in loop SNR of SARA slave due to misalignment of data stream ($R_s = 38.4 \text{ kbps}$; $p_t = .2, .4, .6, .8, 1.0$)

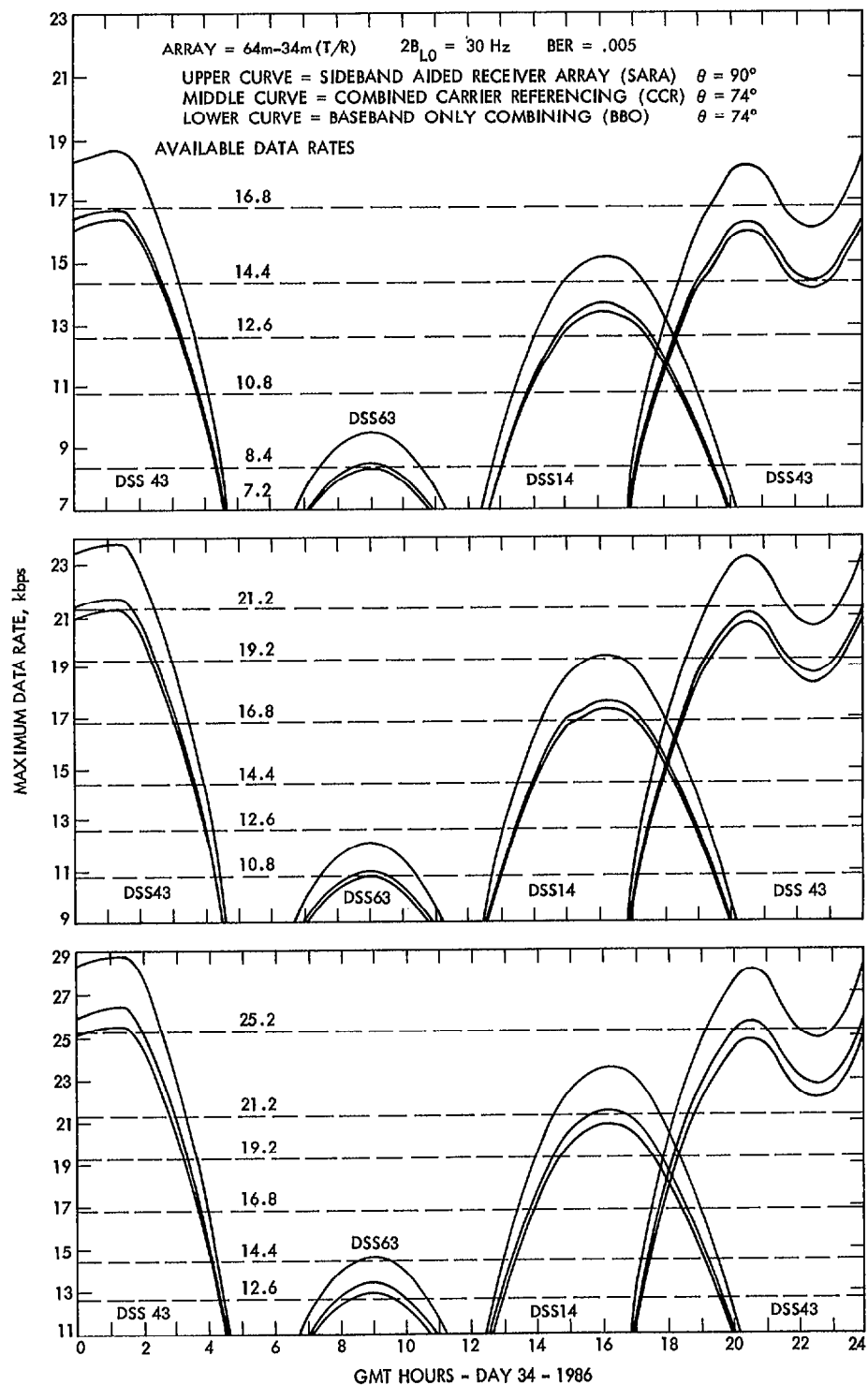


Fig. 6. Data rate vs time: Voyager 2 Uranus encounter

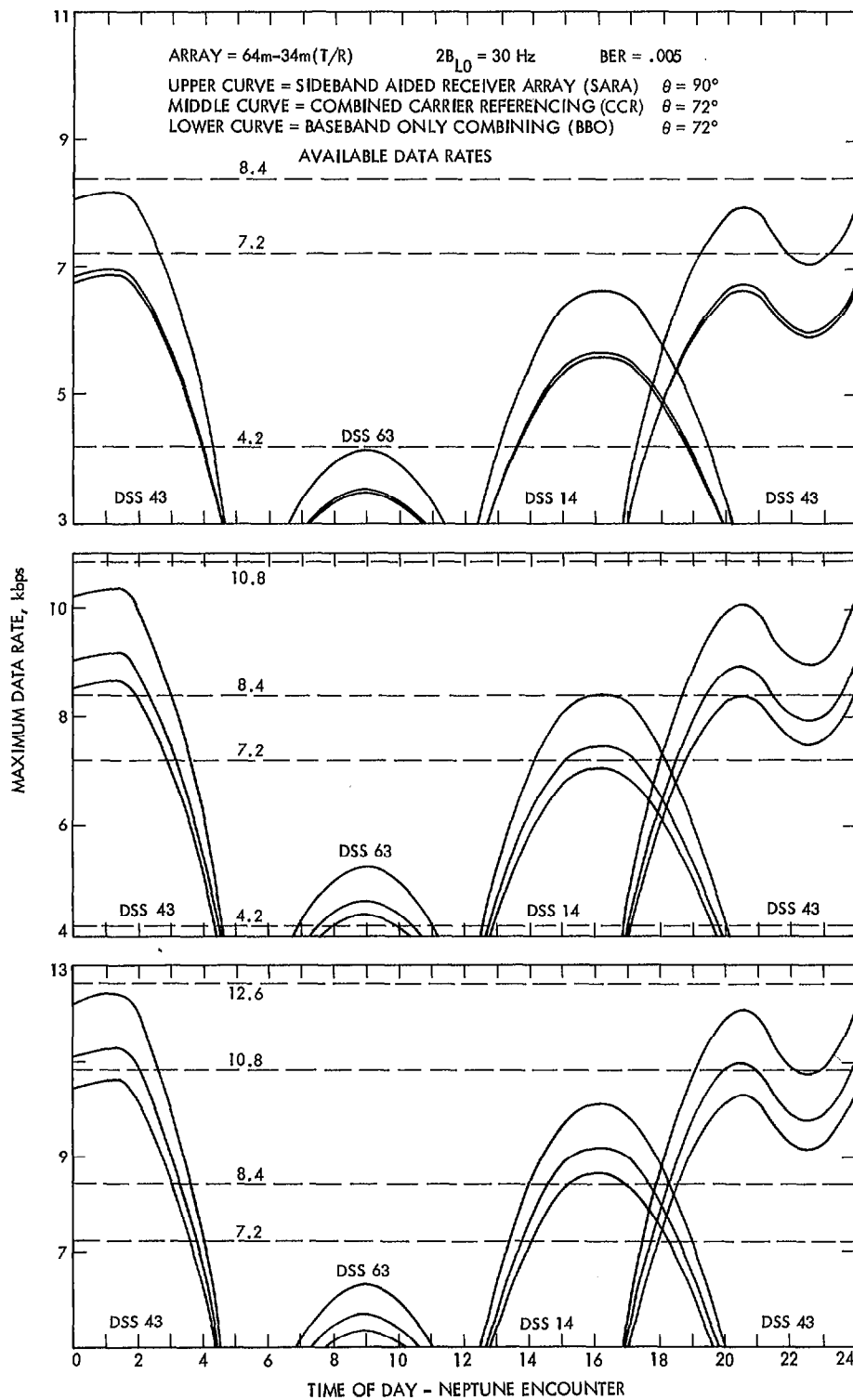


Fig. 7. Data rate vs time: Voyager 2 Neptune encounter



Appendix A

Standard PLL Theory

In a standard phase-locked loop, as shown in Fig. A-1, the input to the loop filter has the form

$$z(t) = A \sin(\phi - \hat{\phi}) + n(t)$$

where ϕ is the phase of the input signal, $\hat{\phi}$ is the loop's estimate of that phase, and $n(t)$ is noise. In our case, $n(t)$ is white Gaussian noise with two-sided spectral density $N_0/2$. If the impulse response of the filter is $f(t)$, then the output of the filter is

$$a(t) = \int_{-\infty}^{\infty} z(x) f(t-x) dx + N(t)$$

where $N(t)$ is the filtered noise. It follows that

$$\frac{d\hat{\phi}}{dt}(t) = K_{VCO} a(t)$$

where K_{VCO} is the frequency gain associated with the VCO. Upon taking Laplace transforms of the equation,

$$s \hat{\phi}(s) = K_{VCO} A F(s) \{ \sin[\phi(s) - \hat{\phi}(s)] + N(s)/A \}$$

where F is the Laplace transform of the function f . For small phase errors, this may be linearized to

$$s \hat{\phi}(s) = K_{VCO} A F(s) [\phi(s) - \hat{\phi}(s) + N(s)/A]$$

whence

$$\hat{\phi}(s) = \frac{K_{VCO} A F(s)}{s + K_{VCO} A F(s)} [\phi(s) + N(s)/A] .$$

The function

$$H(s) = \frac{K_{VCO} A F(s)}{s + K_{VCO} A F(s)}$$

is called the closed-loop transfer function of the loop.

The phase jitter in the loop is given by

$$\sigma^2 = E [\phi - \hat{\phi} - E(\phi - \hat{\phi})]^2$$

where E is the expected value operator. If the phase ϕ of the incoming signal is varying slowly compared to $\hat{\phi}$ (as is the case in SARA), then

$$\begin{aligned} \sigma^2 &= E [E(\hat{\phi}) - \hat{\phi}]^2 \\ &= E (H(s) \phi(s) - H(s) [\phi(s) + N(s)/A])^2 \\ &= E [H(s) N(s)/A]^2 \\ &= \frac{N_0}{2 A^2} \int_{-i\infty}^{i\infty} |H(s)|^2 ds \\ &= \frac{N_0 B_L}{A^2} \end{aligned}$$

where

$$2 B_L = \int_{-i\infty}^{i\infty} |H(s)|^2 ds$$

is the closed-loop bandwidth of the system. The loop signal-to-noise ratio is just the reciprocal of the phase jitter,

$$\rho = \frac{A^2}{N_0 B_L} .$$

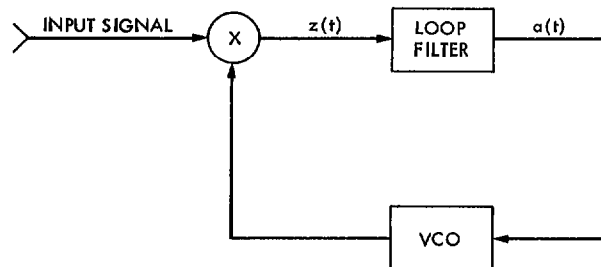


Fig. A-1. A standard phase locked loop (PLL)

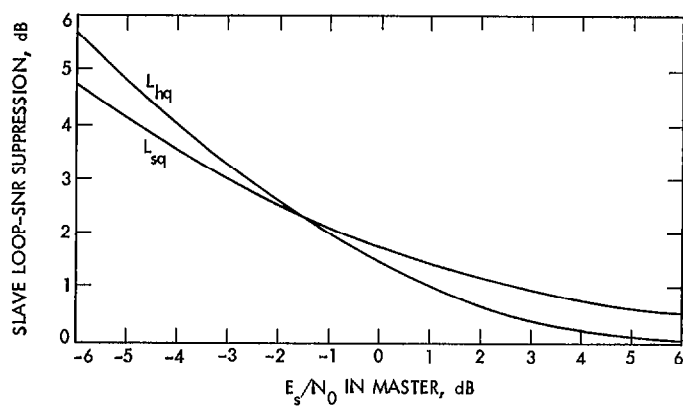


Fig. A-2. Slave loop SNR suppression due to aiding: L_{hq} = suppression with hard-quantization of data estimate; L_{sq} = suppression with soft-quantization of data estimate

Appendix B

Sideband Aiding Without Hard Limiting

In the body of the text, the slave receivers were analyzed assuming a hard-limited estimate of the data from the master was utilized to derive a carrier reference. Since some information is lost in the hard limiting of the output of the data integrator, the question arises as to whether performance could be enhanced if this output itself were used in deriving the slave carrier reference. In the following analysis, all signal references refer to Fig. 2. The incoming signal to the slave has the form

$$x_s(t) = \sqrt{2P_{Ds}} d(t) \cos(\omega_0 t + \phi_s) + N_1(t)$$

where the various quantities have the same meanings as in Eq. (1). The carrier estimate from the VCO is

$$r_s(t) = -\sqrt{2K} \sin(\omega_0 t + \hat{\phi}_s).$$

Neglecting the double frequency components, it follows that

$$y_s(t) = \sqrt{P_{Ds}} K K' d(t) \sin(\phi - \hat{\phi}_s) + N_2(t)$$

where the noise $N_2(t)$ is a white Gaussian process of one-sided spectral density $K K' N_0$ W/Hz over the closed-loop bandwidth. If the data detector box of Fig. 1 does not perform the previously described hard-limiting function, then the signal that is sent to the slave for aiding has the form

$$e(t) = d(t - T_s) + n_3(t)$$

where $d(t)$ can be plus or minus one and $n_3(t)$ is a zero mean Gaussian random variable with variance $N_0/(2E_s)$ over the symbol time T_s .

With proper alignment,

$$z_s(t) = \sqrt{P_{Ds}} K K' \sin(\phi_s - \hat{\phi}_s) + N_3(t) + N_2(t) n_3(t)$$

where $N_3(t) = d(t) N_2(t)$ has approximately the same spectral density as $N_2(t)$ over the closed-loop bandwidth. The other

noise term, $N_2(t) n_3(t)$, has zero mean and spectral density $K K' N_0 [N_0/(2E_s)]$ over the closed-loop bandwidth since $N_2(t)$ and $n_3(t)$ are independent processes. From standard PLL theory (Appendix A), the loop SNR in the slave for this "soft-quantized" implementation is

$$\rho_s^{(sq)} = \frac{P_{Ds}}{N_0 B_{Ls}} \left(1 + \frac{N_0}{2E_s}\right)^{-1}$$

The corresponding loop SNR for the "hard-quantized" implementation of SARA (Eq. 3) is

$$\rho_s^{(hq)} = \frac{P_{Ds}}{N_0 B_{Ls}} \text{erf}^2(\sqrt{E_s/N_0}).$$

The corresponding suppression factors are defined by

$$L_{sq} = \left(1 + \frac{N_0}{2E_s}\right)^{-1}$$

and

$$L_{hq} = \text{erf}^2(\sqrt{E_s/N_0}).$$

These are shown plotted as a function of E_s/N_0 in Fig. A-2. The figure shows that the breakpoint occurs at $E_s/N_0 = -1.55$ dB, with the SARA implementation performing better above this value and the soft-quantized better below this value. The range of values expected for a 64-m antenna arrayed with three 34-m antennas during Voyager Uranus encounter is $-3 \text{ dB} < E_s/N_0 < 0 \text{ dB}$. This brackets the breakpoint. If the SARA implementation is used, the slave loop SNR is degraded by at most 0.3 dB. Since slave loop SNRs at Voyager planetary encounters are expected to be above 14 dB with SARA, this degradation is virtually insignificant. Consequently, the relative difficulties in passing a one-bit (hard-quantized) vs a many-bit (soft-quantized) data estimate to the slave receiver will determine which scheme should be implemented.